Inhoudsopgave

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# 1.0 Phase I – Theory

In this report the behaviour of the following system of differential equations is described:

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| --- | --- |
|  | () |

## 1.1

*Write (1) as a system of first-order differential equations.*

The answer to this is question is fairly easy. It is just a matter of rewriting equation 1 into a vector form (eq. 2):

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|  | () |

## 1.2

*Determine the equilibrium (critical point, stationary point) of the system (3). Apply local linearization in order to investigate the character of the equilibrium (see the supplement). For this one needs to form the Jacobian matrix taken at the equilibrium point. We denote this matrix with A. Further the values of a and b are given. (a = 2 and b = 4.5).*

|  |  |
| --- | --- |
|  | (3) |

First the equilibrium points have to be determined. This will be done by solving the system (4, filled in with values a and b). The equations will be set equal to 0 to determine these points.

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|  | () |

This system only has one equilibrium point (see eq. 4).

To determine the nature of the equilibrium point, the Jacobian has to be determined (4 – I).

|  |  |
| --- | --- |
|  | ( - I) |

The last step is to determine the eigenvalues of the Jacobian (Matrix A in eq. 4 – I). The eigenvalues are computed with the characteristic equation: (3.50 - λ)\*(-4.00 - λ) + 4.00 \* 4.50 = 0.This results in λ=-0.25 ± 1.984\**i* . (eq. 5). The real part is negative this yields in a stable equilibrium point. There also is an imaginary part. This results in a spiral point.

|  |  |  |
| --- | --- | --- |
|  | | (5) |
| **Result:**  λ=-0.25 ± 1.984\**i* |  |  |
|  | (5 – I) |
|  |  |

## 1.3 & 1.4

*Using Python, make a graph in the complex plan containing Λh for h = 0, 0.25, 0.5, 0.75. The scale along the axis must be set according to* ***plt.xlim****(-2.25, 0.25) and* ***plt.ylim****(-2,2). This graph shows eight points in the complex plane. Are these eight points on the line?*

|  |  |  |  |
| --- | --- | --- | --- |
|  | | | Python code: |
| *.* |
|  |
|  | Figure 1 – Stability. |  |  |

The picture above shows seven points. The assignment states that this graph shows eight points. This is because the point (0, 0) is counted twice.

In the graph is clearly visible that the points are in the stability region in h ≤ 0.5. Thus the numeric method is stable for h ≤ 0.5.

## 1.5

*The time step h has to be chosen such that at least 20 steps per period are taken. For which values of h can we achieve this?*

|  |  |  |
| --- | --- | --- |
|  | | (6) |
| **Result:**  *h=0.1583* |  | (6– I) |
|  |

## 1.6

*Of what order in h is the local truncation error of the present numerical method? Present also the order of the global truncation error (discretization error).*

This question can be answered by using Lax’s Theorem.

[Lax’s equivalence theorem (p75)]   
“If a numerical method is stable and consistent, then the numerical approximation converges for the solution for Δt -> 0. Moreover, the global truncation error and the local truncation error are of the same order.“

In this assignment we use the modified Euler method. The local truncation error for this method is well known O(h2) (p74). This means that the global truncation error also is O(h2).

## 1.7

*We can estimate the error in the computed approximation at a certain fixed time t (say t = 1) by comparing the results using time step h and those using time step 2h. Let us denote these results by wh and w2h respectively. The resulting estimation of the error E in wh, i.e. E = y(t) – wh, takes the form*

|  |  |
| --- | --- |
| , for certain α. | () |

*Present the value of α for the present numerical method.*

The estimate of the global truncation-error is (the p of the modified Euler is 2). See also page 79.

|  |  |
| --- | --- |
|  | (7 - II) |

## 1.8

*The supply mechanism for material A may fail sometimes. This means that a = 0. Determine the equilibrium in this case and use local linearization to study the nature of this equilibrium point. Now suppose that at t = t1 the supply fails. How do you think the solution of (3) will behave for t >> t1?*

Determining the equilibrium will be done in the same was as in question 1.2. (a = 0 and b = 4.5)

|  |  |  |
| --- | --- | --- |
|  | | (8) |
| **Result:**  u = 0 ;v ∈ ℝ | or | (8– I) |
| is impossible, so u = 0 and v ∈ ℝ |

This result gives not a finite number of equilibrium points but infinitely many along the v-axis. This means there is an equilibrium line. The Jacobian will be determined for u = 0 and v ∈ ℝ. The phase plane has infinite many equilibrium point on the u-axis.

|  |  |
| --- | --- |
|  | (9 - I) |

|  |  |
| --- | --- |
|  | (9- II) |

For these eigenvalues holds that RE() ≤ 0. So the equilibrium line is stable. When the supply fails at t1 the solution for u and v will converge to a value (for t >> t1 depending on the initial conditions), because the equilibrium is stable.